

Force Limited Random Vibration Test of TESS Camera Mass Model

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ABSTRACT

The Transiting Exoplanet Survey Satellite (TESS) is a spaceborne instrument consisting of four wide field-of-view-CCD cameras dedicated to the discovery of exoplanets around the brightest stars. As part of the environmental testing campaign, force limiting was used to simulate a realistic random vibration launch environment. While the force limit vibration test method is a standard approach used at multiple institutions including Jet Propulsion Laboratory (JPL), NASA Goddard Space Flight Center (GSFC), European Space Research and Technology Center (ESTEC), and Japan Aerospace Exploration Agency (JAXA), it is still difficult to find an actual implementation process in the literature. This paper describes the step-by-step process on how the force limit method was developed and applied on the TESS camera mass model. The process description includes the design of special fixtures to mount the test article for properly installing force transducers, development of the force spectral density using the semi-empirical method, estimation of the fuzzy factor (C^2) based on the mass ratio between the supporting structure and the test article, subsequent validating of the C^2 factor during the vibration test, and calculation of the C.G. accelerations using the Root Mean Square (RMS) reaction force in the spectral domain and the peak reaction force in the time domain.

KEY WORDS: Force limit vibration test, semi-empirical method.

FORCE LIMIT VIBRATION OVERVIEW

Force limit vibration test methodologies were formulated to address the over-testing that commonly occurs during shaker vibration testing. This over-test phenomenon is attributed to the disparity between the mechanical impedance of the test and flight configurations. The shaker (source) exhibits a nearly infinite impedance and as a result responses of the test article are greatly amplified. In contrast, the flight configuration source is typically a softer structure with the impedance significantly lower than that of the shaker interface. As a result the flight system will exhibit coupled dynamic behavior and these coupled-system responses tend to be considerably different from the fixed-base responses of the test article when it is mounted on a shaker.

There are three methods for deriving force limits provided within the NASA-HDBK-7004C¹. They are the simple two degree of freedom system method, the complex two degree of freedom system method, and the semi-empirical method. The semi-empirical method is the most commonly used

of these methods and as it was the method utilized for the testing of the TESS camera mass model, it will be the focus of the proceeding discussion.

The semi-empirical equation relates the amplitude of the force limit to the amplitude of the input acceleration through the following relationships:

$$S_{FF}(f) = C^2 M_o^2 S_{AA}(f) \quad f < f_b \quad (1a)$$

$$S_{FF}(f) = C^2 M_o^2 (f_b/f)^{2n} S_{AA}(f) \quad f \geq f_b \quad (1b)$$

in which:

S_{FF} is the Force Power Spectral Density (FSD)

C^2 is a dimensionless, frequency independent constant

M_o is the mass of the test item

S_{AA} is the input acceleration PSD

f is frequency

f_b is a break frequency

n is a positive constant

The amplitude of the FSD is split into two distinct regions defined by the break frequency, f_b . The break frequency is typically the resonance frequency of the fundamental mode of the test article, where the test article apparent mass is at its peak. After this frequency the apparent mass of the test article rolls off, this is emulated by the value of n in the semi-empirical equation.

Although there is no closed form method to determining the C^2 value, there are a variety of criteria that can be evaluated to determine if the selected C^2 value is appropriate. These criteria include comparing the resultant FSD curve against available interface force test data from similar systems or against limits derived using the TDOF system methods. Additionally, the amount of force clipping, also referred to as the notching in the input acceleration, can be assessed to see if it falls within recommended parameters and the C.G. acceleration of the load can be compared with the quasi-static limit load to ensure correlation. Most recently, vibro-acoustic system analysis has been utilized to provide an insight into determination of force limits. Typically a combination of these criteria are leveraged to determine whether the chosen C^2 value has yielded an appropriate force limit. As a loose guideline, the table below shows the spread of C^2 values that have been used across the industry in the past².

Table 1: Distribution of C^2 values that have been implemented across a variety of test programs².

C^2 Range	All Cases	Lateral Cases	Vertical Cases
$C^2 < 2$	43%	18%	62%
$C^2 < 5$	80%	66%	90%
$C^2 < 10$	92%	82%	100%
$C^2 < 20$	99%	97%	100%
$C^2 < 24$	100%	100%	100%

For the TESS camera module a baseline C^2 value of 6 was selected based on the mass ratio of the source and the load. The camera module weight, not including the mass of the flexures, is approximately 30 lbs and the mass of the instrument is roughly 200 lbs. The chart provided in Figure 1 provides the relationship between the load mass ratio and the C^2 value for various magnitudes of amplification¹. For the TESS camera module mass ratio of 0.15, the corresponding C^2 value is 6.

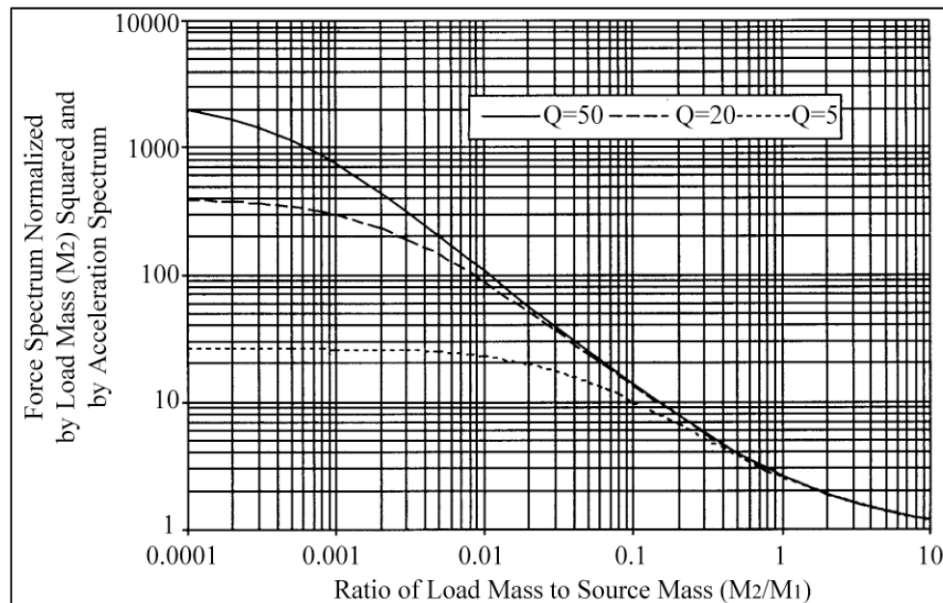


Figure 1: Normalized force specification from simple TDOF system which provides C^2 values on the ordinate and the load mass ratio on the abscissa¹.

TECHNIQUES FOR ANALYZING FORCE LIMITED VIBRATION USING FINITE ELEMENT METHOD

As part of the test program of the TESS camera module a simulated mass mock-up of the camera was manufactured and tested ahead of the actual test hardware in order to provide a low-risk opportunity to gather response data and conduct model correlation studies. In order to generate predictions of the test article acceleration response and report margins of safety a finite element analysis was conducted.

The finite element model was comprised of the following components: test fixture, mounting ring, flexures, simulated mass mock-up, and CBUSH spring elements. The series of CBUSH elements between the mounting ring and the test fixture allow the extraction of the reaction force at that interface; this is vital to the implementation of force limit vibration analysis.

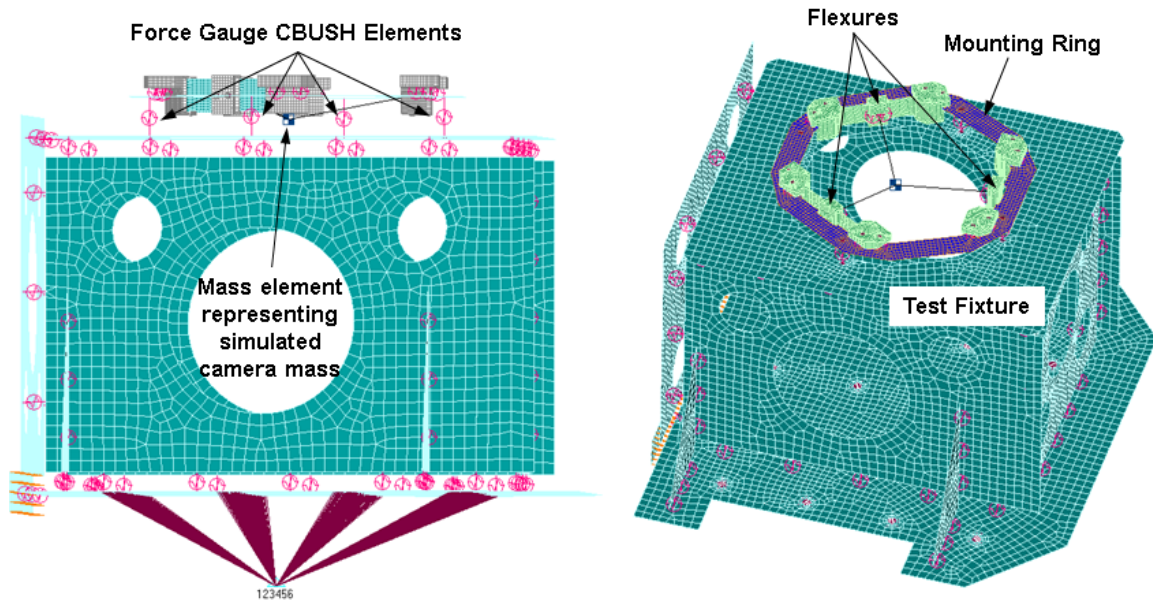


Figure 2: Image of the finite element model of the TESS camera mass model test setup.

Below is the process used to generate force limited input acceleration for the FEM analysis of the TESS camera simulated mass module.

1. Model force gauges as CBUSH elements at the interface between the source and the load
2. Run random vibration analysis using the applicable PSD input
3. Recover the frequency dependent reaction force at the interface between the source and the load using the FORCE card
4. Determine input parameters (C^2 , break frequency, roll-off slope) and substitute into the semi-empirical equation to determine the force limit curve
5. Evaluate where the recovered force exceeds the established force limit specification and “clip” the recovered force
6. Calculate the acceleration input value that will produce the desired reaction force across the entire frequency band using the apparent mass, M_A ($M_A = S_{FF}(f)/S_{AA}(f)$)

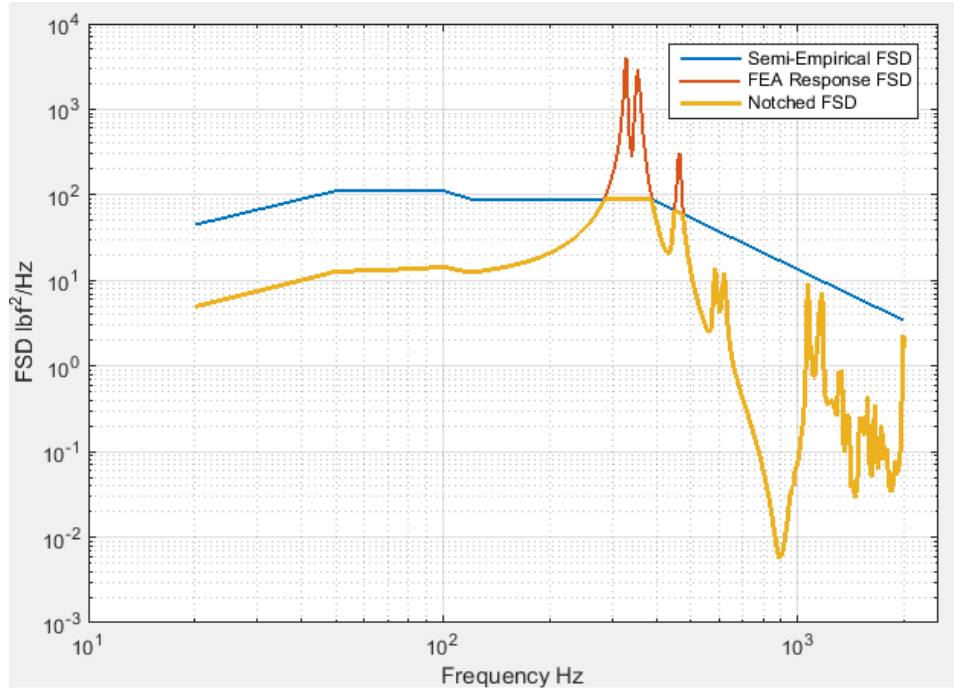


Figure 3: Graph displaying the force response recovered from the CBUSH elements (FEM Response FSD), the force limit curve derived using the semi-empirical equation with $C^2 = 6$, and the resultant force response under the force limit vibration condition (notched FSD). Note that the force response after approximately 450 Hz is indicative of the roll-off in apparent mass.

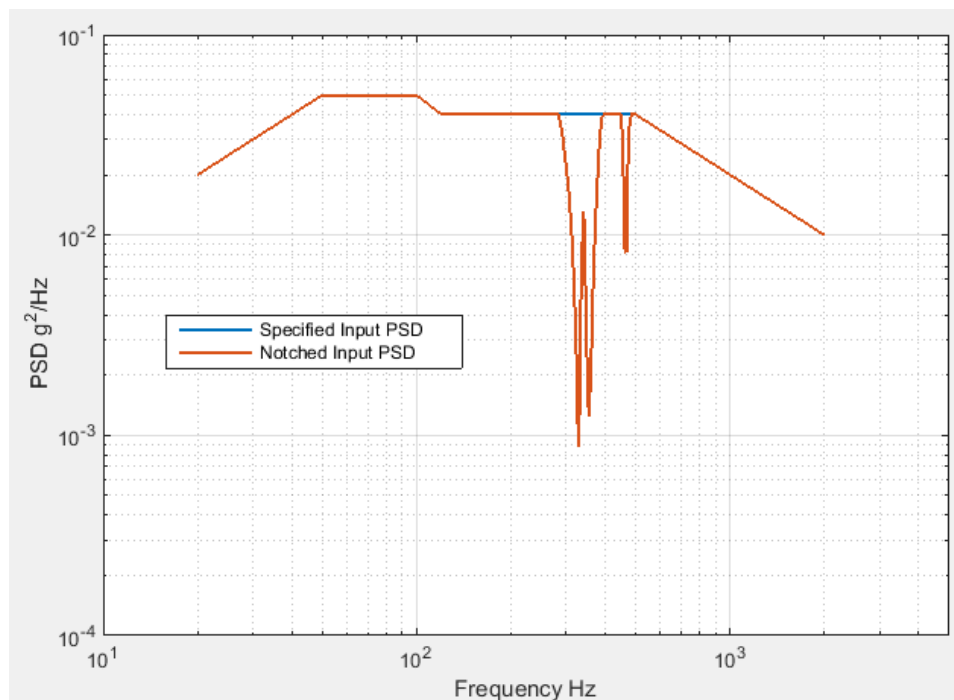


Figure 4: Graph displaying the notched input acceleration PSD generated from the notched FSD curve shown in Figure 3.

TECHNIQUES FOR ESTIMATING FORCE LIMIT VIBRATION ACCELERATION NOTCH DEPTH AND RMS RESPONSE

Although the finite element method for generating force limited input acceleration specifications and producing test article acceleration predictions provides detailed insight into the anticipated test conditions, there are a few simplified methods that can be used to predict force limit vibration testing parameters.

For a single degree of freedom system, Miles equation, shown below, can be used to estimate the RMS response of the system.

$$\sigma_{C.G.} = \sqrt{0.5 * \pi * f_o * Q * S_{AA}(f_o)} \quad (2)$$

in which:

$\sigma_{C.G.}$ is the RMS acceleration of the system

Q is amplification factor at fundamental mode

f_o is frequency of the fundamental mode

S_{AA} is the input acceleration PSD

Miles equation can be modified to account for the effects of force limiting with the use of an empirically derived correction factor, f_r^3 . The formula for calculating the correction factor is based on the ratio of the squares of the amplification factor, Q, and the force limit parameter, C, such that $A^2 = Q^2/C^2$.

$$\sigma_{C.G.} = \sqrt{f_r} * \sqrt{0.5 * \pi * f_o * Q * S_{AA}(f_o)} \quad (3)$$

where

$$f_r = 1 - \frac{2}{\pi} \left(\tan^{-1} \left(\sqrt{A^2 - 1} \right) - \frac{\sqrt{A^2 - 1}}{A^2} \right) \quad (4)$$

Additionally, once the magnitude of this correction factor is calculated the resultant notch depth in input acceleration PSD can be determined using the chart shown in Figure 5.

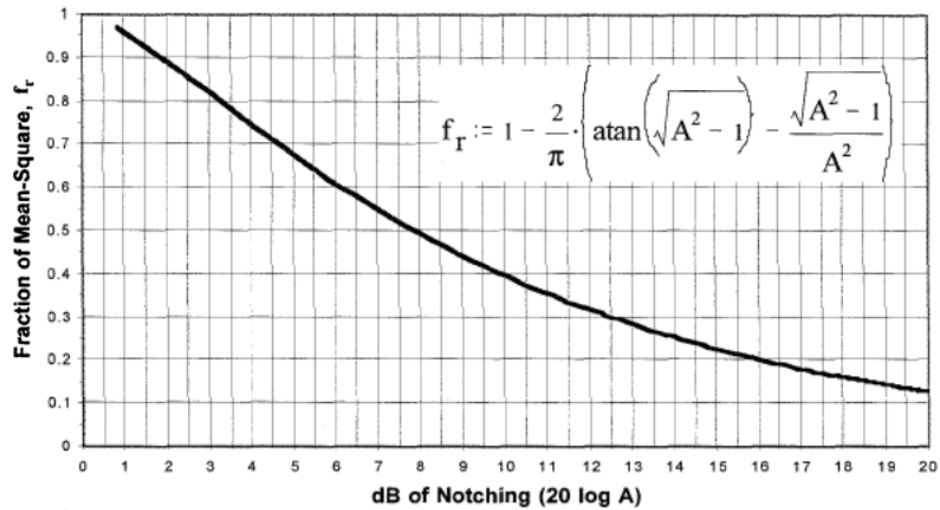


Figure 5: Graph providing the relationship between notch depth and f_r .

Using these formulas the RMS acceleration of the simulated camera mass model can be evaluated. It has been estimated that the fundamental frequency of the TESS camera module simulated mass model will be approximately 325 Hz. At this frequency the random vibration input will be 0.04 g^2/Hz , the assumed Q factor is 50, and the baseline C^2 value is 6. Substituting these values into Equations 3 and 4 results in an estimated response of approximately 8 g_{rms} , which corresponds with a notch depth of 14 dB.

The finite element method discussed previously yielded an estimated response of 12 g_{rms} . As the simulated mass model exhibits multimodal behavior it is not surprising that the modified Miles equation underestimated the RMS acceleration as it assumes a SDOF structure. As shown in Figure 4, the finite element method for deriving force limited input acceleration produced a 14 dB notch at the fundamental frequency, which exactly aligns with the hand calculated estimation.

FORCE LIMITED VIBRATION TESTING EXECUTION AND RESULTS

The TESS camera module simulated mass model was subjected to force limited random vibration testing. In order to conduct this testing, a series of 4 Kistler medium size 9046C4 force gauges were used in tandem with Dytran accelerometers to monitor the reaction force of the mass model and control the input acceleration. In order to provide a thorough data set, time histories of the forces and accelerations were gathered using DataFlex-1000A data acquisition system by DSPCon.

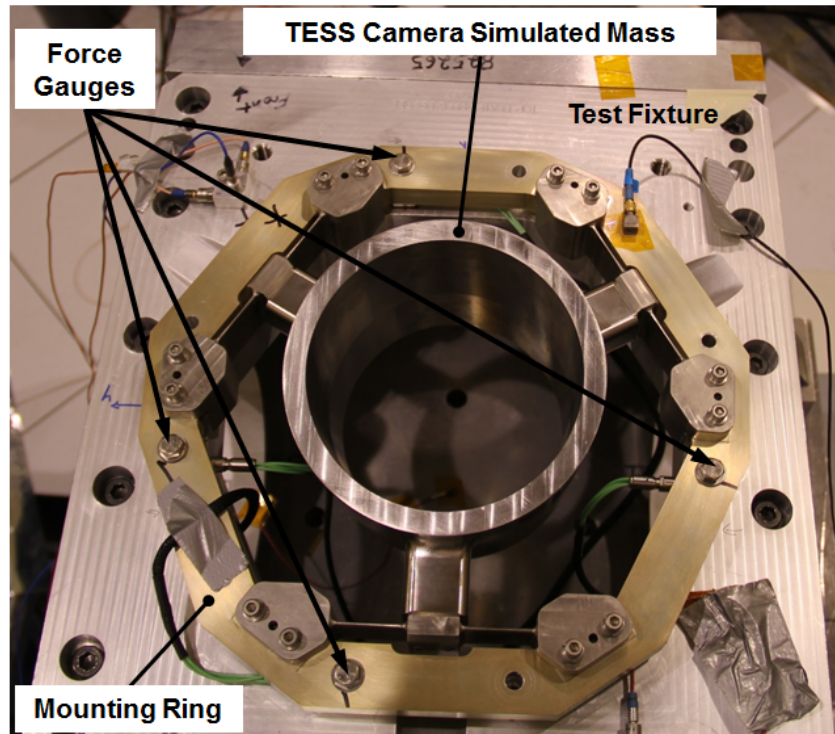


Figure 6: Image of the TESS camera module simulated mass model installed in the test fixture undergoing the vertical (Z-Axis) vibration testing. The test article is fixed to the mounting ring which sits atop the Kistler 9046C4 force gauges.

Below is a summary of the steps to be followed in order to derive the force limit during vibration testing.

1. Run low level random vibration test (ex. -18 dB of full level)
2. Evaluate test article response signatures to determine fundamental frequency (F_n) and roll-off slope
3. Confirm the static mass by calculating the apparent mass, $m(f)$, using the FSD and the input PSD

$$m(f) = \sqrt{\frac{FSD(f)}{PSD(f)}}$$

4. Calculate force limit specification using the semi-empirical equation

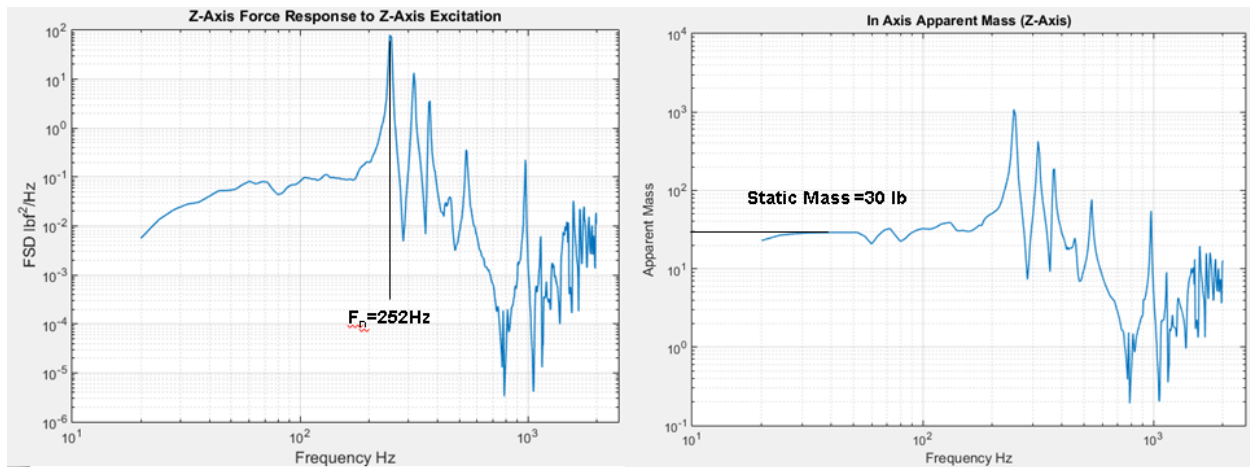


Figure 7: Graph of the TESS camera module simulated mass mock-up force response (L) and associated apparent mass (R).

As previously discussed, for testing of the TESS camera module simulated mass model, C^2 value of 6 was selected based on the mass ratio of the test article to the flight source. During the vibration test, both the acceleration PSD and FSD are used as control parameters; these parameters are plotted as blue curves in Figure 8. As determined from the low level testing, fundamental frequency of the test article in the Z-Axis is 252 Hz. Looking at the plots in Figure 8, the force response is well below the force limit at low frequencies and as such the input acceleration, plotted in red, matches the PSD specification. Around 250 Hz there is distinct notch in the input acceleration curve. The presence of this notch represents that the system response is being force limited, as demonstrated by the fact that the response FSD lies on top of the force limit specification.

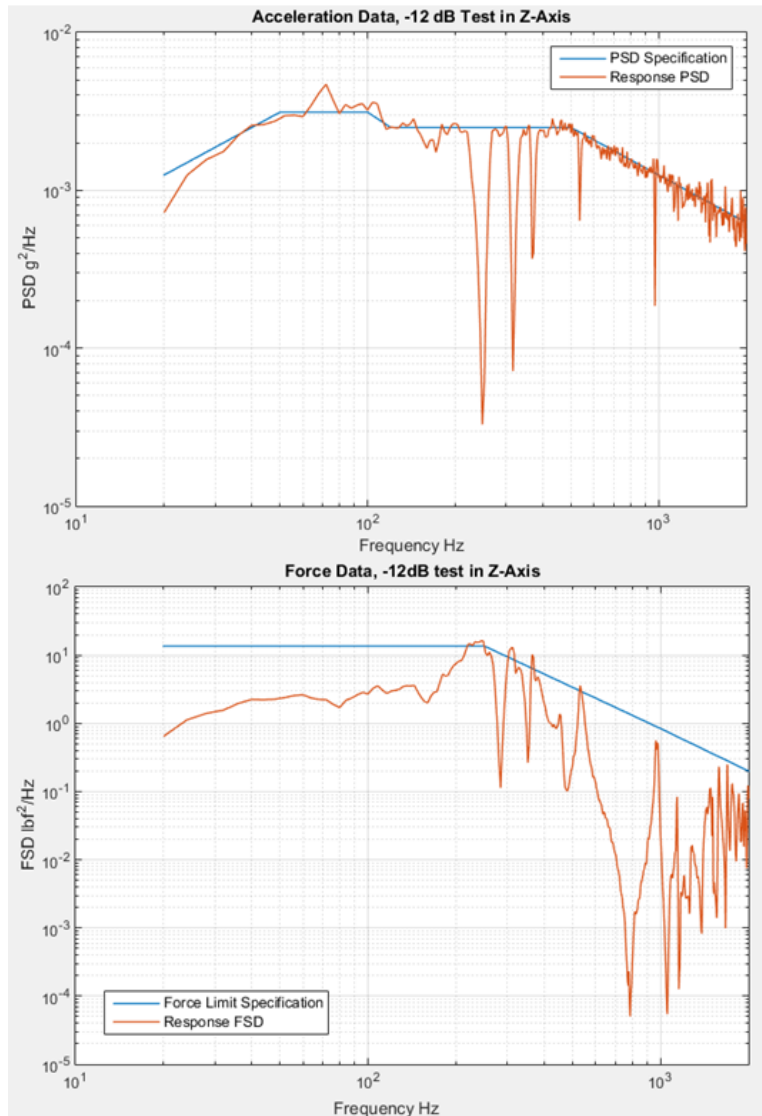


Figure 8: Plots of the force limited input acceleration (notched input PSD) compared to the input PSD specification and the force response compared to the force limit specification.

In accordance with standard practice, the testing was conducted at various levels during the build up to the full level testing. The TESS camera module simulated mass model was tested at -18 dB, -12 dB, and -6dB. Response data was collected at each test level in order to track system linearity. As shown in Table 2, the responses that were tracked during the low level tests demonstrated linear behavior and as a result this test data was utilized to predict full level test responses.

Table 2: TESS camera module simulated mass model response summary.

RMS Summary

Location	Data Chan.	Comp. Resp.	-18 dB Force Limit		-12 dB Force Limit		-6 dB Force Limit		0 dB Level		Units
			RMS response		RMS response		RMS response		Anticipated response		
			X-Axis	Z-Axis	X-Axis	Z-Axis	X-Axis	Z-Axis	X-Axis	Z-Axis	
Control 1		RSS	1.10	0.99	2.16	1.90	4.40	3.45	8.81	17.61	g's
Top Plate	1	X-dir	0.66	0.46	1.28	0.88	2.58	1.58	5.16	10.32	g's
	2	Y-dir	0.66	0.39	1.29	0.74	2.65	1.39	5.30	10.60	g's
	3	Z-dir	0.59	0.79	1.16	1.52	2.39	2.73	4.78	9.56	g's
Control 2		RSS	1.40	0.91	2.73	1.78	5.77	3.82	11.53	7.64	g's
Top Plate	4	X-dir	0.81	0.50	1.58	0.99	3.15	1.81	6.30	3.62	g's
	5	Y-dir	0.61	0.28	1.15	0.52	2.35	1.01	4.70	2.02	g's
	6	Z-dir	0.96	0.71	1.90	1.38	4.22	3.21	8.44	6.42	g's
Response 1		RSS	3.99	3.95	7.17	7.53	8.60	13.85	17.39	27.70	g's
Mounting Ring	7	X-dir	1.73	1.42	3.29	2.72	6.61	5.04	13.22	10.08	g's
	8	Y-dir	1.43	1.22	2.73	2.37	5.38	4.48	10.76	8.96	g's
	9	Z-dir	3.30	3.48	5.75	6.61	1.15	12.10	3.45	24.20	g's
TOTAL INTERFACE FORCE	25	Fx	29.80	0.10	58.50	0.64	116.00	0.71	232.00	1.41	lbf
	26	Fy	13.00	10.20	24.90	20.10	49.90	35.40	99.80	70.80	lbf
	27	Fz	12.40	24.10	23.50	47.20	45.90	80.40	91.80	160.80	lbf

In addition to tracking RMS responses, the peak acceleration and the force responses were tracked using the recorded time history data. The in-axis force response data can be utilized to identify the C.G. acceleration of the test article. From the RMS response the peak acceleration can be estimated using the following equation, assuming a 3-sigma peak event.

$$C.G. Acceleration_{peak} = 3 * \frac{RMS Force}{Static Mass} \quad (5)$$

This estimation can be compared with the true peak C.G. acceleration determined from the time history force response data. Table 3 provides an example of this characterization obtained from the Z-Axis testing of the simulated camera mass. Disparity between the peak acceleration responses estimated from the RMS data versus the time history data suggests the presence of a 5 sigma event.

Table 3: TESS camera module simulated mass C.G. acceleration response.

Static Mass (lbm)	RMS Force Reaction (lbf)	3σ Acceleration Response (g)	C.G. Peak Force Response (lbf)	C.G. Peak Acceleration Response (g)
30	116	11.6	262	18.8

CONCLUSIONS AND RECOMMENDATIONS

As force limit vibration testing helps in achieving the “test like you fly” ideology and reduces the overly conservative design that would result from a standard vibration testing it is highly recommended that this method be utilized for spacecraft programs. As demonstrated throughout

this paper the use of the semi-empirical equation for force limited vibration testing allows for the method to be easily implemented as a part of the analysis and test program of a project.

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BIOGRAPHIES

Alexandra Karlicek is an experienced design, analysis and test engineer specializing in the field of random vibration testing. Alexandra currently works as an Assistant Staff Engineer in Group 74: Structural and Thermal Fluid Engineering at MIT Lincoln Laboratory. Prior to joining MIT LL, Alexandra worked as a structural analyst at Honeywell Aerospace in the Pneumatic Systems Division. She holds a bachelor's degree in Mechanical Engineering from Boston University.

Dr. James Ho-Jin Hwang has over 15 years of experience in the field of dynamic environments and currently works as a subject matter expert in the area of high frequency dynamics and environmental testing at MIT Lincoln Laboratory. Prior to joining MIT LL, Dr. Hwang was a technical lead at Space Exploration Technologies, Jet Propulsion Laboratory, and Raytheon Aircraft Company. Dr. Hwang's area of technical expertise includes simulation of high frequency dynamics using Statistical Energy Analysis (SEA) and environmental tests including high fidelity pyrotechnic shock tests and force limited random vibration tests of complex space structures. He holds a Ph.D. in Aerospace Engineering from the University of Kansas.

Mr. Justin J. Rey has over 13 years of experience in the field of Optomechanical Engineering and is currently the Mechanical Systems Lead for the Transiting Exoplanet Survey Satellite (TESS) Instrument, selected for launch in 2017 as a NASA Explorers program. Mr. Rey serves as a subject matter expert in the area of Integrated Optomechanical Analysis at MIT Lincoln Laboratory. Prior to joining MIT LL, Mr. Rey was Principal Systems Architect for Advanced Active Hybrid Mirror technologies at Northrop Grumman - AOA Xinetics. He holds a M.Sc. in Mechanical Engineering from the University of Southern California and a B.S. in Mechanical Engineering from the University of Rochester.